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Research article

The Solution for Fermat's Last Theorem and Beal's Conjecture using Exponential Algebra Properties

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Abstract

Fermat's Last Theorem (FLT), 1637, states that if $\boldsymbol{n}$ is an integer greater than 2, then it is impossible to find three natural numbers $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{z}$where such equality is met being $(\boldsymbol{x},\boldsymbol{y})>\mathbf{0}$ in $\boldsymbol{x^n}+\boldsymbol{y^n}=\boldsymbol{z^n}$.

Beal's Conjecture (BC), 1993, states that in $\boldsymbol{A^x}+\boldsymbol{B^y}=\boldsymbol{C^z}$ equation, where $(\boldsymbol{A},\boldsymbol{B},\boldsymbol{C},\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})\in\mathbb{Z}^+-\{\mathbf{0}\}$ then $(\boldsymbol{A},\boldsymbol{B},\boldsymbol{C})$ must have a prime factor.

This paper shows the methodology to prove Fermat's Last Theorem and Beal's conjecture using exponential algebra properties showing they cannot result in integer solutions. Since one part of the solution of Beal's Conjecture is based on Fermat's Last Theorem, a short proof of the latter is shown as well.

Key Words: Fermat's Last Theorem, Beal's Conjecture, Exponential algebra properties.

I. INTRODUCTION

FERMAT'S LAST THEOREM

Fermat's last theorem (FLT) or Fermat-Wiles's theorem is one of the most famous theorems in the history of mathematics [1]-[2]-[3]. The unsolved problem stimulated the development of algebraic number theory in the 19th century and the proof of the modularity theorem in the 20th century. It is among the most famous theorems in the history of mathematics and prior to its 1995 proof by Andrew Wiles [4]-[5]-[6] it was in the Guinness Book of World Records for "most difficult mathematical problems".Using modern notation, Fermat's last theorem can be stated as follows:

If n is an integer greater than 2, then you cannot find three natural numbers *x*, *y* and *z* such that equality is met being$(x,y)>0$ in:

$$x^n+y^n=z^n$$

Pierre de Fermat (1667) [7], showed the case of $n=4$, using the infinite descent technique. Alternative proofs of the case $n=4$ were developed late [8]; Leonard Euler (1735), demonstrated the n=3 case confirmed in 1770, [9]-[10]-[11]. Later Germain, [12], stated that if *p* and 2*p*+1 are both primes, then the expression for the power Fermat conjecture *p* meant that one of the *x*, *y* or z would be divisible by *p*. Germain tested for number *n*<100 and Legendre, extended their methods for n<197. Dirichlet and Legendre, (1823-5), [13]-[14], extended the case of n=3 to n=5. More recently, Lame (1840), [15]-[16], provedthe case of n=7. Fermat's Last Theorem has also been proven for the exponents n = 6, 10, and 14, [17]-[18].Only one mathematical proof by Fermat has survived, in which Fermat uses the technique of infinite descent to show that the area of a right triangle with integer sides can never equal the square of an integer. His proof is equivalent to demonstrate that the equation:

$$x^4 - y^4 = z^2$$

has no primitive solutions in integers (no pairwise coprime solutions). In turn, this proves Fermat's Last Theorem for the case n=4, since the equation $a^4 + b^4 = c^4$can be written as $c^4 - b^4 = (a^2)^2$.

The proof of Fermat's Last Theorem in full, for all n, was finally accomplished, however, after 358 years, by Andrew Wiles in 1995 [6], an achievement for which he was honoured and received numerous awards. The solution came in a roundabout manner, from a completely different area of mathematics.

Gerard Frey (1984) called attention to the unusual properties of the some curve as Hellegouarch (1974) [19]-[20], which became called a Frey curve and provided a bridge between Fermat’s Last Theorem and Taniyama-Shimura Conjecture (1955), [21] by showing that a counterexample to Fermat’s Last Theoremwould create such a curve that would not be modular [22]-[23]. Around 1955, the Japanese mathematicians Goro Shimura and Yutaca Taniyama observed a possible link between two apparently completely distinct, branches of mathematics, elliptic curves and modular forms. The resulting modularity theorem (at the time known as the Taniyama–Shimura conjecture) states that every elliptic curve is modular, meaning that it can be associated with a unique modular form. It was initially dismissed as unlikely or highly speculative, and was taken more seriously when number theorist André Weilfound evidence supporting it, but no proof; as a result the "astounding" conjecture was often known as the Taniyama–Shimura-Weil conjecture. It became a part of the Landglands programme, a list of important conjectures needing proof or disproof. Even after gaining serious attention, the conjecture was seen by contemporary mathematicians as extraordinarily difficult or perhaps inaccessible to proof. For example, Wiles’ ex-supervisor John Coates states that it seemed “impossible to actually prove” and Kent Ribet [24], considered himself “one of the vast major of people who believed it was completely inaccessible”, adding that “Andrew Wiles was probably one of the few people on earth who had the audacity to dream that you can actually go and prove it”, [2] pp 191-197.

The conjecture attracted considerable interest when Frey (1986)[25] suggested that the Taniyama-Shimura conjecture (today knows as Taniyama-Shimura Wiles theorem) implies Fermat's Last Theorem. A. Wiles (1995) [6]-[26]-[27]-[28], based in the equation developed by Frey and Taniyama-Shimura Conjecture, showed a general demonstration of Fermat’s Last Theorem, using elliptic curves, schemes of groups, Hecks’ Algebra, Iwasawa theory, Von Neumann-Bernays-Gödel’ theory, Zermelo-Fraenkel’ theory and others complex mathematical tools, all developed many years after Fermat’s lived.

Wiles worked on that task for eight years in near-total secrecy, covering up his efforts by releasing prior work in small segments as separate papers and confiding only in his wife. His initial study suggested proof by induction and he based his initial work and first significant breakthrough on Galois theory before switching to an attempt to extend Horizontal Iwasawa theory for the inductive argument around 1990–91 when it seemed that there was no existing approach adequate to the problem. However, by the summer of 1991, Iwasawa theory also seemed to not be reaching the central issues in the problem. In response, he approached colleagues to seek out any hints of cutting edge research and new techniques, and discovered an Euler system recently developed by Victor Kolyvagin and Matthias Flach which seemed "tailor made" for the inductive part of his proof. Wiles studied and extended this approach, which worked. Since his work relied extensively on this approach, which was new to mathematics and to Wiles, in January 1993 he asked his Princeton colleague, Nick Katz, to check his reasoning for subtle errors. Their conclusion at the time was that the techniques used by Wiles seemed to be working correctly. [2] pp 209-232 and Finally Wiles conclude successfully its demonstration in 1995, [6].

In 2013, J. W. Porras [29] showed another general demonstration using math tools than existed in the 17th century.

This article presents another unpublished solution, different from the successful solutions in [6]-[29] for Fermat’s Last Theorem, using exponential algebra properties and the connection between the Pythagorean’s Theorem and the Fermat’s Last Theorem found in [29] that showsthat Fermat’s Last Theorem cannot result in integer solutions.

BEAL'S CONJECTURE

Beal, (1993), [30]-[31]-[32]-[33]-[34], stated that $\boldsymbol{A^x} + \boldsymbol{B^y} = \boldsymbol{C^z}$ (note that *x*, *y* and *z* are unique exponents.) would have no solution using coprime bases. While working on Fermat's Last Theorem, Andy Beal studied equations with independent exponents. He worked on several algorithms to generate solution sets, but the nature of the algorithms he developed required a common factor in the bases. He suspected that using coprime bases might be impossible and set out to test his hypothesis. With the help of computers and a colleague, Andy Beal tested this for all variable values up to 99. Many solutions were found, and all had a common factor in the bases.

Andy Beal wrote many letters to mathematics periodicals and number theorists. Among the replies were two considered responses from number theorists. Dr. Harold Edwards from the department of mathematics at New York University and author of "Fermat's Last Theorem, a genetic introduction to algebraic number theory" [35], confirmed that the discovery was unknown and called it "quite remarkable". Dr. Earl Taft from the department of mathematics at Rutgers University relayed Andy Beal's discovery to Jarell Tunnell who was "an expert on Fermat's Last Theorem", according to Taft's response, and also confirmed that the discovery and conjecture were unknown. There is no known evidence of prior knowledge of Beal's conjecture and all references to it begin after Andy Beal's 1993 discovery and subsequent dissemination of it. The related ABC conjecture hypothesizes that only a finite number of solutions could exist.

Any solutions to the Beal conjecture will necessarily involve three terms all of which are 3-powerfull numbers, i.e. numbers where the exponent of every prime factor is at least three. What distinguishes Beal's conjecture is that it requires each of the three terms to be expressible as a single power greater than 3. For that reason these examples:

$$271^3 + 2^3 3^5 73^3 = 919^3$$

$$3^4 29^3 89^3 + 7^3 11^3 167^3 = 2^7 5^4 353^3$$

They cannot be considered as a counterexample of the Beal conjecture.

This article corrects errors found in [36] and is confirming that Beal's Conjecture (BC) not only met with prime common bases, but also with composite common bases if one of the exponents is less than three after eliminating the common bases can have solutions in $\mathbb{Z}^+ - \{0\}$[1].Finally, this article eliminates the common bases from BC, that is, if all the exponents are different or two exponents are equal but all exponents are greater than two, shows that there are not solutions in $\mathbb{Z}^+ - \{0\}$.

The proof of this conjecture is based on exponential algebra properties and Fermat's Last Theorem, proven by A. Wiles [6]-[26]-[27]-[28] in 1995 and by J.W. Porras Ferreira [29] in 2013.

II. A SIMPLE SOLUTION OF FERMAT'S LAST THEOREM

A. *Fermat's Last Theorem*

$$z^n = x^n + y^n \Rightarrow x, y > 0 \; and \; coprimes, n > 2, (x, y, n) \in \mathbb{Z}^+ - \{0\}, \qquad z \notin \mathbb{Z}^+ - \{0\}$$

Around 1637, Fermat wrote his Last Theorem in the margin of his copy of the *Arithmetica* next to Diophantus' sum-of-squares problem [2] page 80:

"Cubum autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos, et generaliter nullam in infinitum ultra quadratum potestatem in duos eiusdem nominis fas est dividere cuius rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet."

"It is impossible to separate a cube into two cubes, or a fourth power into two fourth powers, or in general, any power higher than the second, into two like powers. I have discovered a truly marvelous proof of this, which this margin is too narrow to contain"

The margin note became known as Fermat's Last Theorem.

[1] $\mathbb{Z}^+$-$\{0\}$ It means the set of positive integers without zero

It is not known whether Fermat had actually found a valid proof.

Thisproof of FLT is based on another mathematical theorem, which is proved below. This paper also reviews the two solutions described in [29], and uses mathematic tools that existed in the 17TH century, time in which Fermatlived, by which one may think that it might be the “marvelous proof” Fermat was referring when he wrote his famous note and solve the mystery [2]

B. Reviews the two solutions describe in [29]

The Figure 1 summarizes the concept applied in [29], which showed that:

For ${z_{n=2}}^2 = x^2 + y^2 = (y+m)^2$ can have solutions in $(x, y, z, m) \in \mathbb{Z}^+ - \{0\}$.

But in:

$${z_{n>2}}^n = x^n + y^n = (y+q)^n = (y+q)^2(y+q)^{n-2} = {z_{n>2}}^2 {z_{n>2}}^{n-2} \Rightarrow {z_{n>2}}^2 = {x_1}^2 + {y_1}^2$$

Must not have minimum solution where $(x_1, y_1, z_{n>2}) \in \mathbb{Z}^+ - \{0\}$, using the well-ordering principle of the natural numbers and then $z_{n>2} \notin \mathbb{Z}^+ - \{0\}$.

The *well-ordering principle of the natural numbers*in the second sense, is used when that proposition is relied on for the purpose of justifying proofs that take the following form: to prove that every natural number belongs to a specified set S, assume the contrary and infer the existence of a (non-zero) smallest counterexample. Then show either that there must be a still smaller counterexample or that the smallest counterexample is not a counter example, producing a contradiction. This mode of argument bears the same relation to proof by mathematical induction that "If not B then not A" (the style of modus tollens) bears to "If A then B" (the style of modus ponens). It is known light-heartedly as the “minimal” method and is similar in its nature to Fermat’s method of “infinite descent”.

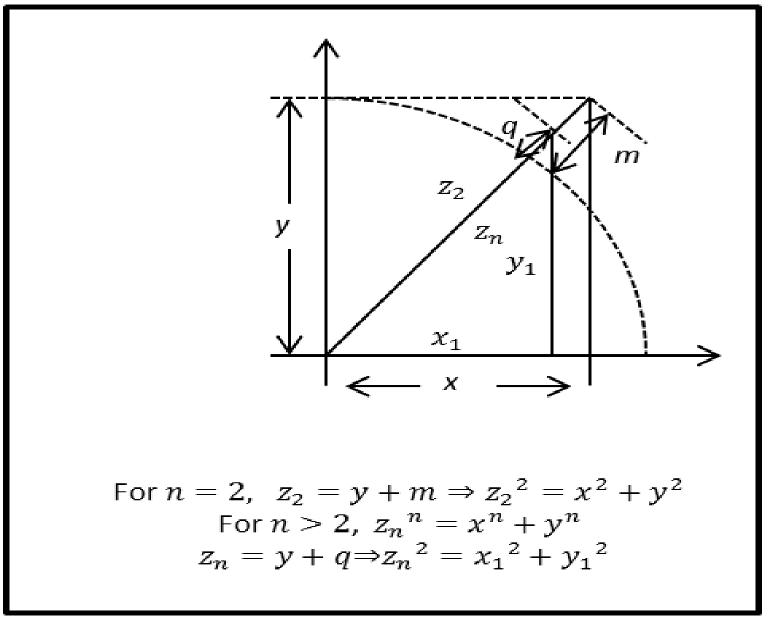

Fig. 1: Graphic representation of ${z_{n=2}}^2 = x^2 + y^2$ and ${z_{n>2}}^n = x^n + y^n$ equations, with $(x, y) \in \mathbb{Z}^+ - \{0\}$ and coprimes.

C. Theorem 1

In equation$x^n + y^n = {z_n}^n$ it is always true that:$z_{n=2} > z_{n>2} \Rightarrow (x, y) \in \mathbb{Z}^+ - \{0\} (x, y)$ coprimes.

Proof:

1. $x^n + y^n = {z_n}^n$ for $n>1$, (x, y) coprimes and $(x, y) \in \mathbb{Z}^+ - \{0\}$

2. Let's $z_{n=2}$ the solution of $z_{n=2}{}^2 = x^2 + y^2$. Let $z_{n>2}$ the solution of $z_{n>2}{}^n = x^n + y^n$ and assuming that $z_{n=2}$ is solution of $z_{n>2} \Rightarrow (x^2, y^2) \in \mathbb{Z}^+ - \{0\}$
3. $z_{n>2}{}^2 = x^2 + y^2 = z_{n=2}{}^2$……… According to 2.
4. $x^n + y^n = z_{n=2}{}^2 z_{n>2}{}^{n-2}$ ………………………………...…According to 3.
5. $x^n + y^n = (x^2 + y^2) z_{n>2}{}^{n-2}$….According to 3 and 4.
6. $x^{n-2}x^2 + y^{n-2}y^2 = z_{n>2}{}^{n-2}x^2 + z_{n>2}{}^{n-2}y^2$ ……………………………………. According to 5.
7. To preserve the equality in 6, the $z_{n>2}{}^{n-2}$coefficient of (x^2, y^2), on the right side of the equation, must be:

   $x^{n-2} < z_{n>2}{}^{n-2} < y^{n-2}$

8. $x < z_{n>2} < y$if $z_{n>2}{}^2 = z_{n=2}{}^2$…… According to 7.
9. $z_{n>2}{}^n = z_{n=2}{}^2 z_{n>2}{}^{n-2} < z_{n=2}{}^2 z_{n=2}{}^{n-2} = z_{n=2}{}^n$…………………………………..According to 8.
10. $z_{n=2} > z_{n>2}$ ……………….…...… According to 9.
11. Q.E.D.[2]

Corollary: In the equation:

$$z_n{}^n = x^n + y^n \text{for } n \geq 1$$

it is true that: $z_{n=1} > z_{n=2} > z_{n=3} > z_{n=4} > z_{n=5} \ldots (x, y) \in \mathbb{Z}^+ - \{0\}$

Examples:

a. With $(x, y, z_{n=2}) \in \mathbb{Z}^+ - \{0\}$

   $z_n{}^n = 3^n + 4^n \Rightarrow z_{n=1} = 7 > z_{n=2} = 5 > z_{n=3} = 4{,}4779 \ldots > z_{n=4} = 4{,}2845 \ldots > z_{n=5} = 4{,}1740 \ldots > \cdots$

b. With $(x, y) \in \mathbb{Z}^+ - \{0\}$ and $z_{n=2} \notin \mathbb{Z}^+ - \{0\}$

   $z_n{}^n = 2^n + 3^n \quad \Rightarrow \quad z_{n=1} = 5 > z_{n=2} = 3{,}6055 \ldots > z_{n=3} = 3{,}2710 \ldots > z_{n=4} = 3{,}1382 \ldots > z_{n=5} = 3{,}0751 \ldots > \cdots$

Let proof Fermat's Last Theorem using the Pythagorean theorem and Theorem 1.

The traditional interest in Pythagorean triples connects with the Pythagoream Theorem [37] in its converse form, it states that a triangle with sides of lengths *a*, *b*, and *c* has a right angle between the *a* and *b* legs when the numbers are a Pythagorean triple. Examples of Pythagorean triples include (3, 4, 5) and (5, 12, 13). There are infinitely many such triples [38] and methods for generating such triples have been studied in many cultures, beginning with the Babylonians [39] and later ancient Greek, Chinese, and Indian mathematicians [40]. Fermat's Last Theorem is an extension of this problem to higher powers, stating that no solution exists when the exponent 2 is replaced by any larger integer.

D. Theorem 2

Fermat's Last Theorem:

$$z_n{}^n = x^n + y^n \Rightarrow x, y > 0 \text{ and coprimes}, n > 2, (x, y, n) \in \mathbb{Z}^+ - \{0\}, \qquad z_n \notin \mathbb{Z}^+ - \{0\}$$

Proof:

1. Let $x^2 + y^2 = z_{n=2}{}^2$, $(x, y) \in \mathbb{Z}^+ - \{0\}$ and (x, y) coprimes, where $z_{n=2}$ is the hypotenuse and (x, y) the other two sides of a right triangle. (Figure 1).
2. Let $z_n{}^n = x^n + y^n$ where : $z_{n=2} > z_{n=3} > z_{n=4} > z_{n=5}$ ...as proven in Theorem 1 above. (Figure 1)
3. There are two cases to be analyzed for $x^2 + y^2 = z_{n=2}{}^2$ and $z_{n>2}{}^n = x^n + y^n$ equations:

[2] From latín - Quad Eran Demonstrandum -

a. First case:

In $x^2 + y^2 = {z_{n=2}}^2$ equation, according to Pythagorean's Theorem, [37]-[38]-[39]-[40]-[41]-[42]-[43]-[44], if $(x, y, z_{n=2}) \in \mathbb{Z}^+ - \{0\}$ and (x, y) coprimes, must be generated Primitive Pythagorean Triples $(x, y, z_{n=2}) \in \mathbb{Z}^+ - \{0\}$.

Based on Theorem 1:

$\boldsymbol{z_{n=2}} > \boldsymbol{z_{n>2}}$ therefore it is impossible to have other integer solutionsfor $(x_1, y_1, z_{n>2}) \in \mathbb{Z}^+ - \{0\}$ inside of the right triangle of figure 1. By definition, [35]-[36], the shape of the triangle with Primitive Pythagorean Triple is the smaller with (x, y) sides, $z_{n=2}$ hypotenuse and $(x, y, z_{n=2}) \in \mathbb{Z}^+ - \{0\}$, then with $z_{n>2} < z_2$ it is impossible to have others $(x_1, y_1, z_{n>2}) \in \mathbb{Z}^+ - \{0\}$ integer solutions, with the same triangle shape but smaller, therefore $z_{n>2}$is not resolving in $\mathbb{Z}^+ - \{0\}$.

Proof of first case:

To have the same triangle shape in the Figure 1, (x, y) must be divided by the same a number: $x_1 = \frac{x}{a}$ and $y_1 = \frac{y}{a}$ but (x, y) are coprimes, then x_1, y_1 or both must not be integers, therefore $z_{n>2}$is not resolving in $\mathbb{Z}^+ - \{0\}$.

If a is irrational $x_1 = \frac{x}{a}$ and $y_1 = \frac{y}{a}$ will be irrational and $z_{n>2} = \sqrt{{x_1}^2 + {y_1}^2} = \frac{\sqrt{x^2+y^2}}{a} = \frac{z_{n=2}}{a}$ is irrational because a natural number divided by an irrational number the result is an irrational number then:

$z_{n>2}$is not resolving in $\mathbb{Z}^+ - \{0\}$.

b. Second case:

In $x^2 + y^2 = {z_{n=2}}^2$ equation, (x, y) coprimes and $z_{n=2} \notin \mathbb{Z}^+ - \{0\}$, then $z_{n=2}$ is irrational number. ($z_{n=2}$ comes from $\sqrt{x^2 + y^2}$).

Based on Theorem 1:

$\boldsymbol{z_{n=2}} > \boldsymbol{z_{n>2}}$ therefore it is impossible to have any Primitive Pythagorean Triple with the same shape of Figure 1 with $(x_1, y_1, z_{n>2}) \in \mathbb{Z}^+ - \{0\}$, if (x, y) coprimes, therefore $z_{n>2}$is not resolving in $\mathbb{Z}^+ - \{0\}$.

Proof of second case:

To have the same triangle shape in the Figure 1, (x, y) must be divided by the same a number: $x_1 = \frac{x}{a}$ and $y_1 = \frac{y}{a}$ then x_1, y_1 or both must be not integers, therefore $z_{n>2}$is not resolving in $\mathbb{Z}^+ - \{0\}$.

If a is integer or rational numbers then $z_{n>2} = \sqrt{{x_1}^2 + {y_1}^2} = \frac{\sqrt{x^2+y^2}}{a}$ will be irrational because $\sqrt{x^2 + y^2}$ is irrational and $z_{n>2}$is not resolving in $\mathbb{Z}^+ - \{0\}$

If a is irrational $x_1 = \frac{x}{a}$ and $y_1 = \frac{y}{a}$ will be irrational and $z_{n>2}$is not resolving in $\mathbb{Z}^+ - \{0\}$

4. Fermat's Last Theorem is proven ………………………………………….……… According to 3.

Q.E.D.

III. Beal's Conjecture Solution

A. *Beal's Conjecture*

The Beal Conjecture (1993), states that $A^x + B^y = C^z$ equation, where $(A, B, C, x, y, z) \in \mathbb{Z}^+ - \{0\}$ then (A, B, C) must have a prime factor.
In other words:
$If\ a^r + b^s = c^t$, coprimes a, b, c, $(r, s, t) > 2$, there are no solutions in $\mathbb{Z}^+ - \{0\}$.

C. Giraldo and J. W. Porras [36] in July 2013 presented a solution of the Beal's Conjecture. Two mistakes were found on [36], which this article resolved. The mistakes were:

a. Point (5), page 750 [36], is not valid. There is no need for u and v to be integers always.
b. When it is assumed that u and v are rational numbers, pages 751 [36]. That is impossible.

The following solution provides a solution to Beal's Conjecture,with those errors corrected.

B. *Theorem 3*

$If\ a^r + b^s = t$, coprimes a, b, c, $(r, s, t) > 2$, there are no solutions in $\mathbb{Z}^+ - \{0\}$.

Demonstration:

1. According to Beal's conjecture: $A^x + B^y = C^z$, where $(A, B, C, x, y, z) \in \mathbb{Z}^+ - \{0\}$ then (A, B, C) must have a prime factor:

   $A^x|p^n, B^y|p^n, C^z|p^n \Rightarrow p$ is a prime number and $n \geq 1$ (the symbol | means that p^n divides exactly to A^x, B^y and C^z)

2. $A^x = p^n a^r, B^y = p^n b^s, C^z = p^n c^t \Rightarrow (r, s, t) \in \mathbb{Z}^+ - \{0\}$, ……….………………………........ According to 1.

3. If at least one of the exponents is less than three after the elimination of prime factor, then $a^r + b^s = c^t$ equation can have $(a, b, c, r, s, t) \in \mathbb{Z}^+ - \{0\}$. (See Table I).

TABLE I

Equation examples for $A^x + B^y = C^z$, $\Rightarrow (A, B, C, x, y, z) \in \mathbb{Z}^+ - \{0\}$, (A, B, C) with a common prime factor and $a^r + b^s = c^t$ where one of the exponents is less than three after the elimination of prime factor and $(a, b, c, r, s, t) \in \mathbb{Z}^+ - \{0\}$.

A^x	B^y	C^y	Prime factor	a^r	b^s	c^t
3^6	18^3	3^8	$p^n = 3^6$	1	2^3	3^2
8^3	28^3	6^4	$p^n = 2^2$	2^5	7^2	9^2

Beal's Conjecture also complies when the common factor is a composite number, if after the elimination of the composite factor one of the exponents is less than three. For example, the equation $a^2 + b^4 = c^3$ have integer solutions and can be converted to Beal's Conjecture, with generating numbers (a, b, c) [45] of the form[3]:

$$a = (3m^4 + 4n^4)(9m^8 - 408m^4n^4 + 16n^8)$$
$$b = 6mn(3m^4 - 4n^4)$$
$$c = 9m^8 + 168m^4n^4 + 16n^8$$

This form has infinite solutions.

[3] Rafael Parra Machío. ECUACIONES DIOFÁNTICAS, pp 22. Web: http://hojamat.es/parra/diofánticas.pdf

Table II shows some examples with $(m = 1, n = 1)$ and $(m = 3, n = 2)$, where a^{12k} for $k \geq 1$ can be a common factor of A^x, B^y and C^z with $A^x = a^{2+12k}, B^y = (a^{3k}b)^4, C^z = (a^{4k}c)^3$ and then $(A, B, C, x, y, z) \in \mathbb{Z}^+ - \{0\}$.

A^{14}	B^4	C^3	Composite numbers	a^2	b^4	c^3
2681^{14}	$(2681^3 \cdot 6)^4$	$(2681^4 \cdot 193)^3$	2681^{12}	2681^2	6^4	193^3
142946261^{14}	$(142946261^3 \cdot 6444)^4$	$(142946261^4 \cdot 280873)^3$	142946261^{12}	142946261^2	6444^4	280873^3

4. Assuming: $a^r + b^s = c^t$, $with$ coprimes a, b, c, $(r, s, t) > 2$, there are not solutions in $\mathbb{Z}^+ - \{0\}$

5. Assuming that r is the smaller exponent and greater than two:

   $$(a^r + b^s = c^t) \equiv \left(a^r + \left(b^{s/r}\right)^r = \left(c^{t/r}\right)^r\right) \Rightarrow \text{(exponent property)}$$

6. There are three cases to be analyzed for $b^{s/r}$ and $c^{t/r}$

7. **First case $s | r$ and $t | r$ or $(b = {b_1}^{kr}$ and $s \nmid r)$ or $(c = {c_1}^{kr}$ and $t \nmid r)$, $(b_1, c_1, k) \in \mathbb{Z}^+ - \{0\})$:**
   $b^{s/r} = u,\ c^{t/r} = v, \quad (u, v) \in \mathbb{Z}^+ - \{0\}$

   According to points 5 and 6:

   $$(a^r + b^s = c^t) \equiv (a^r + u^r = v^r)$$

   This equation has no solution in $\mathbb{Z}^+ - \{0\} r > 2$: Fermat's Last Theorem (A. Wiles 1995, [6]-[26]-[27]-[28] J. W. Porras Ferreira 2013, [29] and Theorem 2).

8. **Second case $s \nmid r$ or $t \nmid r$ and $(b \neq {b_1}^{kr}$ or $c \neq {c_1}^{kr}, (b_1, c_1, k) \in \mathbb{Z}^+ - \{0\})$; therefore: $b^{s/r}$ or $c^{t/r}$ or both must not be rational numbers:**

   Proof:
   Assuming $b^{s/r} = \frac{d}{e}$ or $c^{t/r} = \frac{f}{g}$ or both are rational $(d, e, f, g) \in \mathbb{Z}^+ - \{0\} \Rightarrow (d, e)$ coprimes and (f, g) coprimes:
   $\left(\frac{d}{e}\right)^r$ or $\left(\frac{f}{g}\right)^r$ or both are not integers. It is impossible that an integer be the same than a rational number; therefore: $b^{s/r}$ or $c^{t/r}$ or both must not be rational numbers.

9. **Third case $s \nmid r$ or $t \nmid r$ and $(b \neq {b_1}^{kr}$ or $c \neq {c_1}^{kr}, (b_1, c_1, k) \in \mathbb{Z}^+ - \{0\})$; $b^{s/r}$ or $c^{t/r}$ or both must be irrational numbers**:
   If any $b^{s/r}$ or $c^{t/r}$, or both bases are irrational numbers, then the equation:

   $(a^r + b^s = c^t) \equiv a^r + \left(b^{s/r}\right)^r = \left(c^{t/r}\right)^r$ is not resolving in $\mathbb{Z}^+ - \{0\}$

10. The demonstration scheme is the same for any smaller exponents and it is not necessary to expand the demonstration for each of the others exponents if one of them is the smallest.

11. According to 5, 6, 7, 8, 9 and 10 the assumed in 4 is correct, then the BC is demonstrated.

Q.E.D.

Corollary one: $a^r + b^s = c^t$, coprimes a, b, c, can has solution in $\mathbb{Z}^+ - \{0\}$**,** if one of the exponent $(r, s, t)<3$

Corollary two: $a^r + b^s = c^t$, coprimes a, b, c, has no solution in$\mathbb{Z}^+ - \{0\}$**,** if all the exponents(r, s, t)>2 and two of them are equal.

To clarify, Figure 2 shows how it is possible to go from Beal's Conjecture to the $a^r + b^s = c^t$ equation or from $a^r + b^s = c^t$ to Beal's Conjecture with solutions in $\mathbb{Z}^+ - \{0\}$ if one of the exponents (r, s, t) is less than three.

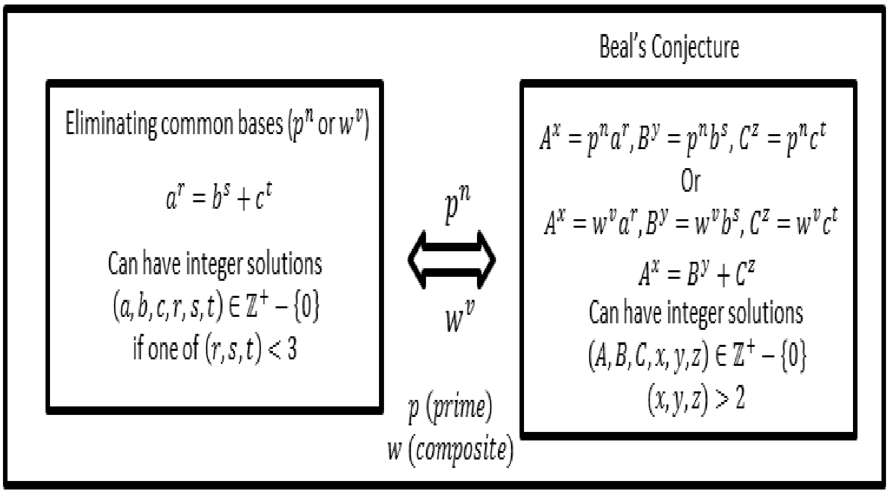

Fig. 2: How is it possible to go from BC to the $a^r + b^s = c^t$ equation with solutions in $\mathbb{Z}^+ - \{0\}$ or from $a^r + b^s = c^t$ to BC with solutions in $\mathbb{Z}^+ - \{0\}$ if one of the exponents (r, s, t) is less than three.

IV. CONCLUSION

Using the demonstration of Fermat's Last Theorem as a base and using mathematical transformations and exponent properties, it was possible to confirm Beal's Conjecture. The equation $A^x + B^y = C^z$, where $(A, B, C, x, y, z) \in \mathbb{Z}^+ - \{0\} \Rightarrow (x, y, z) > 2$, has common bases, (prime or composite numbers) and after eliminating the common bases, it is shown that when the resulting equation has one of the exponents less than three, it can have integer solutions. If the resulting equation or in any equation that have all the exponents (r, s, t) greater than two (all different, all equal or just two exponents being equal), with (a, b) coprimes bases, applying the properties of exponents, confirms that the equation:

$$a^r + b^s = c^t,\ (a, b) \text{ coprimes and} (r, s, t) > 2, \text{there are not solutions in} \mathbb{Z}^+ - \{0\}.$$

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Conflict of interest

I don't have any conflict of interest, since this work is the product of the research done by my rely on the references described in the manuscript and my mathematical knowledge on the subject of the article.

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